Critical points for the functional $E(u) = \int |\Delta u|^2$ are called biharmonic maps and are natural fourth order analogues of harmonic maps. Compactness theory for harmonic maps in two dimensions is well understood. In this talk we will discuss recent work with T. Lamm in which we determine energy quantization and the $C^0$ limit picture for sequence of approximate biharmonic maps from four dimensional manifolds into spheres. In particular, when the approximate map is in $L \log L$ we demonstrate that the energy does not concentrate. Moreover, we determine that if the $L \log L$ norm of the approximate maps does not concentrate, the image of the bubbles are connected without necks. (Received August 25, 2015)