We discuss the Martin compactification of a complete non-compact surface of negative curvature, which is bounded from below but approaches zero along some paths to the infinity (of the surface). In particular, we investigate positive eigenfunctions with eigenvalue one of the Laplace operator and prove a certain uniqueness result. The uniqueness problem arises in our study of deformations of three-dimensional collapsed gradient steady Ricci solitons. (Received September 14, 2015)