Given a commutative ring $R$ and two $R$-modules $N \subseteq M$, we say that $N$ is relatively divisible pure (RD-pure) in $M$ (or that $N$ is an RD-pure submodule of $M$) if for every $r \in R$, $rM \cap N = rN$; i.e., every element $n \in N$ that is divisible by some $r \in R$ in $M$ is already divisible by $r$ in $N$. Let $D$ be an integrally closed domain and $A$ a torsion-free $D$ algebra. We say that $a \in A$ is RD-pure if for all $d \in D$ we have $dA \cap D[a] = dD[a]$; that is, if $D[a]$ is an RD-pure $D$-submodule of $A$.

Our motivation for this topic comes from working with companion matrices and attempting to isolate properties of them that may be useful in other algebras. RD-purity is one such property, because any companion matrix in the matrix algebra $M_n(D)$ is RD-pure (although they are not the only RD-pure elements of $M_n(D)$) and RD-purity has a surprising number of equivalent definitions in a general $D$-algebra $A$. In addition to the definitions given above, RD-pure elements of $A$ can be characterized in terms of their minimal polynomials over $D$, their null ideals over residue rings of $D$, or polynomials integer-valued at elements of $A$. In this preliminary report, we will discuss these definitions and related musings. (Received January 13, 2017)