Let $k$ be a field. We associate a quasi-hereditary algebra $R(M)$ over $k$ to any matroid $M$, such that the Ringel dual of $R(M)$ is $R(M^*)$, where $M^*$ is the dual matroid. The representation theory of this algebra is closely related to certain bilinear forms introduced by Schechtman-Varchenko and Brylawski-Varchenko. Like the classical Schur algebras, our algebras are semisimple over fields of characteristic zero. In this generic setting, our algebras can be used to categorify a formula of Kook-Reiner-Stanton. Our work is motivated by the geometry of symplectic resolutions and, in particular, hypertoric varieties. (Received January 17, 2017)