In this talk, we will define tau-functions given as matrix elements for the action of $\hat{GL}_n$ on $n$-component Fermionic Fock space. We will explain how to show that the tau-functions for the $n = 2$ case satisfy the $A_\infty/2$ $Q$-system. Since $Q$-systems are of interest in various places in mathematics, for example in combinatorics and in representation theory, it is natural to expect that the tau-functions for the $n > 2$ cases also satisfy interesting systems of difference equations. We will discuss the difference equations satisfied by the $n = 3$ tau-functions and will give conjectures for the general $n$ case. We will conclude the talk by discussing the progress we have made in analyzing these systems of equations. If time permits, a generalization of this work will be discussed. (Joint with Maarten Bergvelt) (Received January 16, 2017)