Springer varieties are a family of subvarieties of the flag variety that are parametrized by partitions. They arise in a prototypical example of a geometric representation: their cohomology carries a natural action of the symmetric group, the top-dimensional cohomology of each Springer variety is an irreducible representation, and all irreducible representations of the symmetric group can be obtained this way.

Schubert varieties are a family of subvarieties of the flag variety that are parametrized by permutations. They form a CW-decomposition of the flag variety and so induce a module basis for the cohomology of the flag variety. They are also closely related to other important combinatorial objects, including Schur functions.

For each Springer variety, we describe a union of Schubert varieties whose Betti numbers agree with the Betti numbers of the Springer variety. We also discuss some natural combinatorial and geometric conjectures and questions that emerge from this work. (Received January 20, 2015)