Given an algebraic curve $C$ of genus $g$ defined over $\mathbb{C}$, we say a point $P$ is a Weierstrass point if $\dim(\mathcal{L}(gP)) > 1$, where $\mathcal{L}(D)$ is the Riemann-Roch space associated to a divisor $D$. One can generalize this to define a higher-order Weierstrass point (which we call a $q$-Weierstrass point, for $q \geq 1$), and one can also talk about the weight of a Weierstrass point. For any curve of genus $g > 1$ and any $q \geq 1$, there are a finite number of $q$-Weierstrass points with bounded weights.

The subject of Weierstrass points is an interesting one with immediate applications. In particular, the set of Weierstrass points is an invariant of a curve which is useful in studying the automorphism group of a curve.

In this talk, we will consider Weierstrass points on superelliptic curves, which are curves of the form $y^n = f(x)$ for $f(x)$ a separable polynomial of degree $d$. Under a mild hypothesis, it is well known that the branch points of such curves are Weierstrass points. We will investigate the weights of these branch points for given values of $n$ and $d$ and use them to obtain some asymptotic results. We will also discuss arithmetic properties of these points. (Received January 20, 2015)