In the 1970s, D. Alekseevskii conjectured that if $G/K$ is a non-compact, homogeneous Einstein space with negative scalar curvature, then $K$ is a maximal compact subgroup of $G$. In this talk, we will present the current state of knowledge, with evidence both for and against the conjecture. (Received January 15, 2015)