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**Anne Greenbaum\*** ([greenbau@uw.edu](mailto:greenbau@uw.edu)), University of Washington, Applied Math Dept., Box 353925, Seattle, WA 98195. *A New Proof that Any Disk Containing the Numerical Range is a 2-Spectral Set.*

Let  $A$  be an  $n$  by  $n$  matrix and let  $W(A) = \{\langle Aq, q \rangle \in \mathbb{C} : \|q\|_2 = 1\}$  denote its numerical range. In 1975, Okubo and Ando showed that if  $W(A)$  is a subset of the unit disk  $\mathbb{D}$ , then  $\mathbb{D}$  is a 2-spectral set for  $A$ ; that is, for any polynomial  $p$ ,  $\|p(A)\| \leq 2\|p\|_{\mathbb{D}}$ , where the norm on the left is the operator 2-norm, or, the largest singular value of  $p(A)$ , and  $\|\cdot\|_{\mathbb{D}}$  on the right denotes the infinity norm:  $\sup_{z \in \mathbb{D}} |p(z)|$ . Crouzeix has conjectured that  $W(A)$  itself is a 2-spectral set for  $A$ , and very recently Palencia and Crouzeix [<https://arxiv.org/abs/1702.00668>] were able to prove that  $W(A)$  is a  $(1 + \sqrt{2})$ -spectral set for  $A$ . We use the Palencia-Crouzeix result, along with a new result about the function  $\hat{f}$  that maximizes  $\|f(A)\|/\|f\|_{W(A)}$  over all  $f$  analytic in  $W(A)$ , to give a new proof that any disk containing  $W(A)$  is a 2-spectral set for  $A$ . We discuss some classes of matrices for which the  $1 + \sqrt{2}$  bound in the Palencia-Crouzeix paper can be reduced to 2. (Received February 22, 2017)