A matricial view of the Karpelevič Theorem.

The question of the exact region in the complex plane of the possible single eigenvalues of all $n$-by-$n$ stochastic matrices was raised by Kolmogorov in 1937 and settled by Karpelevič in 1951 after a partial result by Dmitriev and Dynkin in 1946. The Karpelevič result is unwieldy, but a simplification was given by Djoković in 1990 and Ito in 1997. The Karpelevič region is determined by a set of boundary arcs each connecting consecutive roots of unity of order less than $n$.

However, noticeably absent in the Karpelevič theorem (and the above-mentioned works) are realizing-matrices (i.e., a matrix whose spectrum contains a given point) for points on these arcs. In this talk we show that each of these arcs is realized by a single, somewhat simple, parameterized stochastic matrix. Other observations are made about the nature of the arcs and several further questions are raised. The doubly stochastic analog of the Karpelevič region remains open, but a conjecture about it is amplified.

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