We establish $L^p$ bounds for the Bourgain-Stein spherical maximal operator in the setting of compactly supported Borel measures $\mu, \nu$ satisfying natural local size assumptions $\mu(B(x, r)) \leq Cr^s\mu, \nu(B(x, r)) \leq Cr^s\nu$. Taking the supremum over all $t > 0$ is not in general possible for reasons that are fundamental to the fractal setting, but we are able to obtain single scale ($t \in [1, 2]$) results. As an application, we consider to what extent it is possible to obtain a lower bound on the dimension of a set from a lower bound on the dimension of the intersection of this set with a suitably large collection of manifolds. (Received January 21, 2016)