Simon K Donaldson*, sdonaldson@scgp.stonybrook.edu. *Survey of progress and problems on $G_2$-manifolds.

There are two “exceptional” cases in Berger’s classification of Riemannian holonomy groups. These occur in dimensions 7 and 8 and are related to the Cayley numbers. In this talk we will discuss the 7-dimensional case of $G_2$-manifolds, with holonomy the exceptional Lie group $G_2$. The subject began in 1987 when Bryant found local examples. Nine years later, Joyce constructed compact examples by resolving the singularities of quotients of tori. Another construction of compact examples, with input from complex algebraic geometry, was found by Kovalev and later substantially extended by Corti, Haskins, Nordstrom and Pacini. This body of examples opens up many intriguing but difficult questions on existence and moduli of solutions. The main aim of the talk will be to give an overview of this area. We will also discuss adiabatic limits of fibred $G_2$-manifolds, and compare with related work on other geometric structures, such as Calabi-Yau manifolds. (Received February 02, 2016)