

1122-05-270

**Gavin D King\*** (gking3@uwyo.edu). *Searching for Schurian Balanced Sets*. Preliminary report.

Let  $S$  be a set of vectors on the unit sphere in  $\mathbb{R}^d$  coming from a primitive idempotent of a symmetric association scheme. For some vectors  $u, v \in S$  and constants  $x, y \in \mathbb{R}$ , let  $R_{x,y}(u, v)$  be the sum of all vectors  $s \in S$  such that  $\langle s, u \rangle = x$  and  $\langle s, v \rangle = y$ . Such a set  $S$  is said to have the *balanced set property* of Terwilliger if for all  $u, v \in S$  and  $x, y \in \mathbb{R}$ ,

$$R_{x,y}(u, v) - R_{y,x}(u, v) \parallel y - x$$

Balanced sets arise naturally in the study of association schemes, where an association scheme gives rise to a balanced set if and only the *representation diagram* of that idempotent is a tree. If the representation diagram is a path, we say that it has the  $Q$ -polynomial property; several examples of these schemes are known. Only one non-degenerate scheme is known whose representation diagram is a non-path tree: the 4-dimensional polytope known as the 600-cell. One special subcategory of association schemes is Schurian schemes, which arise from the orbitals of generously transitive permutation groups. In this talk, we describe the search for Schurian schemes giving balanced sets, both in the much more abundant  $Q$ -polynomial case and the much more poorly understood “ $Q$ -tree” case. (Received August 15, 2016)