The gap, which is the positive difference of the smallest two distinct energies of a quantum system, is fundamental in quantum many-body physics. Often in physics, gapless systems are very special; they require high symmetry such as Goldstone modes and continuous symmetry breaking, or they require emergent gauge structure like in gapless quantum spin liquids. From a complexity perspective, the gap problem is formidable—just deciding whether the gap goes to zero or not is provably undecidable. Here we prove that the lack of an energy gap is a completely generic property. Specifically we prove that quantum local Hamiltonians, whose local terms are independent matrices from standard random matrix ensembles or certain random projectors, are gapless. The Hamiltonian can be on a lattice in any spatial dimension or on a graph with a bounded maximum vertex degree. The proofs combine quantum information techniques with ideas from analysis, probability and random matrix theories to explicitly construct gapless examples with dense open neighborhoods. Gapped systems are expected to be less resourceful. Therefore, in addition to pushing the undecidable subset into a low-dimensional corner, our results suggest that quantum Hamiltonians are typically richer than expected. (Received July 14, 2016)