Bootstrap percolation is a cellular automaton introduced in 1979 by Chalupa, Leath, and Reich. Fix $r \geq 2$. In $r$-neighbor bootstrap percolation on a graph $G$, all vertices are either “infected” or “uninfected”. In this process, an initially infected set $A \subseteq V(G)$ grows by iteratively infecting all uninfected vertices with at least $r$ infected neighbors. If all vertices eventually become infected, we say that the initial set $A$ is $r$-contagious.

Let $m(G, r)$ denote the minimum size of an $r$-contagious set in $G$. Clearly, $m(G, r) \geq \min\{|V(G)|, r\}$. What conditions on $G$ imply that $m(G, r) = r$? Let $\sigma_2(G) = \min\{d(x) + d(y) : xy \not\in E(G)\}$. Freund, Poloczek, and Reichman recently showed that if $\sigma(G) \geq n$, then $m(G, 2) = 2$. We show that in fact, $\sigma_2(G) \geq n - 2$ is almost enough to imply $m(G, 2) = 2$: if $\sigma_2(G) \geq n - 2$ and $m(G, 2) > 2$, then either $G$ is in one of four infinite families or $G$ is one of nine exceptional graphs.

We also show that if $G$ is a graph with degree sequence $d_1 \leq \cdots \leq d_n$ such that for all $1 \leq i < n/2$, either $d_i \geq i + 1$ or $d_{n-1} \geq n - i - 1$, then either $m(G, 2) = 2$, $G \cong C_5$, or $G$ is in one of two infinite families. (Received August 29, 2016)