Combinatorics of the asymmetric simple exclusion process and Koornwinder polynomials.

The asymmetric simple exclusion process (ASEP) is a process from statistical physics that describes the dynamics of interacting particles hopping right and left on a one-dimensional finite lattice with open boundaries, with parameters $\alpha$, $\beta$, and $q$ describing the hopping rates. One can compute the steady state probabilities of the ASEP as sums over combinatorial objects such as alternative tableaux.

The two-species ASEP is a generalization in which there are two species of particles, heavy and light. Only the heavy particles are able to enter and exit at the left and right of the lattice and with rates $\alpha$ and $\beta$, respectively. Two adjacent particles of different species can swap with rate 1 if the heavier particle is on the left, and rate $q$ if it is on the right. We give a combinatorial formula for the steady state probabilities of the two-species ASEP by introducing the rhombic alternative tableaux.

Recently, a fascinating connection was discovered between a more general 5-parameter two-species ASEP and Koornwinder-Macdonald polynomials. We introduce rhombic staircase tableaux and provide combinatorial formulae for the 5-parameter two-species ASEP, and consequently for moments of Koornwinder-Macdonald polynomials. (Received August 29, 2016)