We initiate a theory of noncommutative Möbius functions. The departing point is a classical result of Solomon which states that the algebra of a lattice is split-semisimple and provides an expression for the primitive idempotents of the algebra in terms of the Möbius function of the lattice. We consider left regular bands, a certain class of idempotent semigroups which arise in connection to real hyperplane arrangements. Such semigroups may be regarded as “noncommutative lattices”. Geometry motivates the introduction of objects such as faces, flats, and lunes. Flats and lunes constitute the objects and morphisms of a category. Its incidence algebra contains two distinguished affine subspaces of equal dimension, one of noncommutative zeta functions and the other of noncommutative Möbius functions. The two are in bijection under inversion. The primitive idempotents of the semigroup algebra admit an explicit expression in terms of these noncommutative Möbius functions. Solomon’s result, and classical facts about Möbius functions, are recovered when the semigroup is commutative (and hence a lattice). (Received August 30, 2016)