Hecke studies the distribution of fractional parts of quadratic irrationals with Fourier expansion of Dirichlet series. This method is generalized by Behnke and Ash-Friedberg, to study the distribution of the number of totally positive integers of given trace in a general totally real number field of any degree. When the field is cubic, we show that the asymptotic behavior of a weighted Diophantine sum is related to the structure of the unit group. The main term can be expressed in terms of Grössencharacter $L$-functions. (Received August 26, 2016)