Let $X$ be a non-singular projective hypersurface of degree $d$ and dimension $n$. When $n+2$ is a multiple of $d$, say $n+2 = (r+1)d$, the first non-vanishing hodge number in dimension $n$ is $h^{r,n-r} = 1$. If we work over the finite field of $q$ elements, and write $Z(X,T)$ for the zeta function of $X$, it is known that $(Z(X,T) \prod_{i=0}^{n}(1-q^iT))^{(-1)^{n+1}}$ is a polynomial for general $X$ with a (distinguished) unique reciprocal root $u_X$ satisfying $\text{ord}_q(u_X) = r$. In this work, we describe a formula for this unique reciprocal zero in terms of a distinguished $p$-adic solution to a particular $A$-hypergeometric system. (Received August 22, 2016)