The Moy-Prasad filtrations of $p$-adic groups play an important role in the representation theory of $p$-adic groups. We will introduce the Moy-Prasad filtration and its quotients and indicate how they form representations over the residue field. We will then present new descriptions of these Moy-Prasad filtration representations that are in some sense independent of the residue field characteristic. As an application, we will explain how these results lead to new supercuspidal representations. (Received August 28, 2016)