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Beltrami fields with non-constant proportionality factor via moving frames. Preliminary report.

A Beltrami field on an open set in \mathbb{R}^3 is a vector field \mathbf{u} satisfying the conditions

$$\operatorname{curl} \mathbf{u} = f\mathbf{u}, \quad \operatorname{div} \mathbf{u} = 0$$

for some function f . When f is constant, local solutions depend on 2 arbitrary functions of 2 variables. For non-constant f , Enciso and Peralta-Salas have shown that solutions are rare; in fact, there are no solutions at all unless f satisfies an explicit differential equation.

In this work, we study Beltrami fields via a moving frames approach. For local solutions near any point where $\nabla f \neq 0$ we show that:

(1) If the level surfaces of f are open subsets of planes or spheres, there are no solutions unless the level sets of f are contained in parallel planes or concentric spheres, in which case solutions depend on 2 functions of 1 variable.

(2) Otherwise, there is at most a 3-dimensional space of solutions.

Unfortunately, the question of precisely which functions f admit solutions remains computationally intractable. A Cartan-Kähler argument suggests that such functions f should depend on 3 functions of 2 variables, so the question remains: which functions f admit solutions, and how many solutions does each such function admit? (Received August 23, 2016)