Polygon iterations, which can be thought of as discrete flows on the space of polygons, provide an abundance of interesting discrete dynamical systems in geometry, especially in Euclidean and affine geometries. Recently, the advance of computers has allowed the study of polygon iterations in projective geometry, which was previously limited by the high computational complexity of the associated rational maps, to take off. Notable examples are the pentagram map and the projective midpoint map, both first studied by Richard Schwartz as potential analogues of the classical midpoint map.

In this talk, we will discuss recent work on a one-parameter family of projectively natural polygon iterations that includes both the pentagram map and the projective midpoint map. They can be regarded as autonomous discrete dynamical systems on the non-compact space of polygons, modulo projective transformations. Except for 2 parameters, corresponding to the pentagram map and its inverse, these polygon iterations are observed to possess a single globally attracting fixed point, which allows us to define their Julia sets. Coincidentally, when observing the varying Julia sets, we discovered that this family contains two projective analogues of Varignon’s theorem for quadrilaterals. (Received August 29, 2016)