Much effort has been devoted to generalizing the Calderón–Zygmund theory from Euclidean spaces to metric measure spaces, or spaces of homogeneous type. Here the underlying space $\mathbb{R}^n$ with Euclidean metric and Lebesgue measure is replaced by a set $X$ with general metric or quasi-metric and a doubling measure. Further, one can replace the Laplacian operator that underpins the Calderón–Zygmund theory by more general operators $L$ satisfying heat kernel estimates.

I will present recent joint work with P. Chen, X.T. Duong, J. Li and L.X. Yan along these lines. We develop the theory of product Hardy spaces $H^{p}_{L_{1},L_{2}}(X_{1} \times X_{2})$, for $1 \leq p < \infty$, defined on products of spaces of homogeneous type, and associated to operators $L_{1}$, $L_{2}$ satisfying Davis–Gaffney estimates. This theory includes definitions of Hardy spaces via appropriate square functions, an atomic Hardy space, a Calderón–Zygmund decomposition, interpolation theorems, and the boundedness of a class of Marcinkiewicz-type spectral multiplier operators. (Received August 07, 2016)