We study the injectivity radius or thickness function $r$ on the configuration space $C(N, S^2)$ of $N$ distinct points on the unit 2-sphere. Criticality for maximizing $r$ is equivalent to the existence of a balanced contact graph of geodesic arcs whose vertices are (a subset of) the points in the configuration (often, but not always, the remaining points are “rattlers”). We also develop a Morse Lemma for the second order behavior of $r$ near such a critical configuration: $r = q + p + o(2)$, where $q$ is a quadratic function on the tangent space of $C(N, S^2)$, and where $p$ is piecewise linear and concave. In general, such critical configurations comprise a semi-algebraic subvariety of $C(N, S^2)$ and the corresponding critical values are a finite subset of the interval $[\pi/N, \pi]$. For small values of $N$, we describe all the critical configurations and the corresponding Morse Complex; we also aim to understand special values of $N$, like $N = 12$, where some surprises occur. (This is part of a joint project with W. Kusner, J. Lagarias and S. Shlosman.) (Received August 29, 2016)