In 2007, Marcos Zarzar suggested that algebraic surfaces $X$ over a finite field $\mathbb{F}_q$ with small Picard number (the rank of the Néron-Severi group over the finite field) might be used to produce good evaluation codes. His key idea was that limiting the Picard number of $X$ puts restrictions on the irreducible curves on the surface that can appear as components of divisors in the hyperplane section divisor class (and this is important since reducible divisors often yield codewords of small weight in the associated evaluation codes). We study this idea and evaluate its potential. In particular, we point out that the sectional genus $g$ of the surface also plays a key role and the cases of $g = 0, 1$ seem to be more favorable than higher $g$. We find bounds on the minimum distance in situations where we can control the irreducible components of the hyperplane sections. We also present several examples of such codes with minimum distance better than the best known examples in Grassl’s tables. (Received August 12, 2016)