Several types of the isomorphism classes of graph coverings have been enumerated by many authors. In 1988, Hofmeister enumerated the double covers of a graph, and this work was extended to $n$-fold coverings of a graph by the second and third authors. For regular coverings of a graph, their isomorphism classes were enumerated when the covering transformation group is a finite abelian or dihedral group in SIAM J. Discrete Math., 11 (1998), 273-285. In this paper, we enumerate the isomorphism classes of graph coverings when the covering transformation group is a $\mathbb{Z}_2$-extension of a cyclic group, including generalized quaternion and semi-dihedral groups. (Received July 21, 2017)