Associated to a finite cyclic subgroup $G$ of $SL_2(\mathbb{C})$, there is a family of noncommutative algebras $O^\tau = O^\tau(\mathbb{C}^2//G)$ representing a universal deformation of the coordinate ring of the classical Kleinian singularity $\mathbb{C}^2//G$.

Earlier, in his thesis, F. Eshmatov constructed an isomorphism between the moduli space of rank one projective modules (noncommutative line bundles) over $O^\tau$ and a certain class of Nakajima quiver varieties $M^\tau$ associated to $G$ via the McKay correspondence. He showed that the varieties $M^\tau$ carry a natural action of the automorphism group $\text{Aut}[O^\tau]$ of the algebra $O^\tau$, and the above isomorphism is equivariant under this action. In this talk, we will prove that the action of $\text{Aut}[O^\tau]$ on $M^\tau$ is actually transitive, and will use this result to give a geometric classification of algebras Morita equivalent to $O^\tau$. We will also compute the Picard group of auto-equivalences of the abelian category of $O^\tau$-modules.

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