For many years, my collaborators and I tried to understand the Coulomb branches of certain field theories from physics and failed miserably. Luckily, recent work of Braverman-Finkelberg-Nakajima gives a mathematical construction of these spaces, and algebras quantizing them. I’ll discuss an approach to the representation theory of these algebras (building on joint work with Braden-Licata-Proudfoot and many other authors). Applications include a version of the Koszul duality between the Higgs and Coulomb branches of such a theory, a new perspective on category O for Cherednik algebras, and a new understanding of coherent sheaves on Coulomb branches. (Received July 11, 2017)