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Numann Malik* (numann@math.brown.edu). *Linearization of the Gross-Pitaevskii Equation around a Vacuum State and the Black Soliton: Low Frequency Effects in 1D*. Preliminary report.

Consider the cubic defocusing NLS

$$i\partial_t u(t, x) + \partial_x^2 u(t, x) - 2(|u(t, x)|^2 - 1)u(t, x) = 0,$$

subject to the non-vanishing boundary conditions $|u(t, x)| \rightarrow 1$ as $|x| \rightarrow \infty$.

Linearizing around $\tanh(x)$ yields the evolution equation

$$\partial_t \begin{bmatrix} w \\ \bar{w} \end{bmatrix} = -i\mathcal{L} \begin{bmatrix} w \\ \bar{w} \end{bmatrix} + O(w^2)$$

where

$$\mathcal{L} = \begin{bmatrix} -\partial_x^2 + 2 & 2 \\ -2 & \partial_x^2 - 2 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} \text{sech}^2(x)$$

and $w = w(t, x) = u(t, x) - \tanh(x)$ is a complex-valued perturbation. We explicitly calculate a formula for the propagator $e^{-it\mathcal{L}}$ obtained from explicit squared Jost solutions.

In the case of linearizing around the vacuum state 1, we end up with \mathcal{L} minus the local terms. The propagator consists of a regular Fourier transform with a singularity at zero frequency. When considering the case of the black soliton we instead get a distorted Fourier transform modulo projection, arising from the presence of an end-point resonance embedded in the continuous spectrum. Analyzing different space-time regions we describe the long-time asymptotics for both cases. (Received July 25, 2017)