We present preconditioned steepest descent solvers for some fourth- and sixth-order nonlinear elliptic equations that include p-Laplacian terms. The highest and lowest order terms are constant-coefficient, positive linear operators. Instead of solving the nonlinear systems directly, we minimize the convex energies associated with the equations. By using the energy dissipation property, we derive a discrete $\ell^p$ bound for the solution, as well as the upper-bound for the second derivative of the energy. These bounds allow us to investigate the convergence properties of our method. In particular, the geometric convergence rate is shown for the nonlinear preconditioned steepest descent (PSD) iteration applied to the regularized equation, which provides a much sharper theoretical result over the existing works. Numerical simulations for thin film epitaxy with slope selection are carried out to verify well-known physical scaling laws for the long time coarsening process. Moreover, we present, for the first time, numerical simulations for the 6-Laplacian thin film epitaxy and the $H^{-1}$ gradient flows of squared phase field crystal model, using the proposed method. (Received June 11, 2016)