Novel strange chaotic attractor theory and constructions for discrete dynamical systems generated by $C^1$ maps $f : U \rightarrow \mathbb{R}^n$, where $U$ is an open subset of $\mathbb{R}^n$ are presented. They are for the most part generalizations and variations of recent results obtained in collaboration with D. Blackmore (Strange attractors for asymptotically zero map, *Chaos, Solitons & Fractals* **68** (2014)). In particular, we describe and analyze generalized attracting horseshoes and generalized attracting multi-horseshoes, which give rise to strange chaotic attractors in any finite-dimensional euclidean space, and are such that their fractal dimensions can be approximated with considerable accuracy. We shall show, for example, that the theory of these types of attractors can be applied to obtain a much shorter proof of the existence of Hénon attractors than that of Benedicks & Carleson. Finally, we shall indicate how the new constructions may lead to methods for extending the rank-one theory of Wang & Young to attractors of rank greater than one. (Received September 11, 2016)