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Peter Nyikos J. Nyikos* (nyikos@math.sc.edu), Department of Mathematics, University of South Carolina, Columbia, SC 29208. *Locally compact spaces: ω_1 -compactness vs. σ -countable compactness.* Preliminary report.

A *space of countable extent*, also called an ω_1 -compact space, is one in which every closed discrete subspace is countable. Obviously, every σ -countably compact space [i.e., the union of countably many countably compact spaces] is ω_1 -compact. The addition of local compactness makes the converse sensitive to set-theoretic independence results. For example:

Theorem 1 [Lyubomir Zdomsky]. If P-Ideal Dichotomy (PID) holds and $\mathfrak{p} > \aleph_1$, then every locally compact, ω_1 -compact space of cardinality \aleph_1 is σ -countably compact.

Theorem 2. In $\text{MM}(\mathfrak{S})[\mathfrak{S}]$ models, every locally compact, hereditarily normal, ω_1 -compact space is σ -countably compact.

If we replace “hereditarily normal” by the stronger “monotonically normal” in this theorem, then we only need the consequence PID of $\text{MM}(\mathfrak{S})[\mathfrak{S}]$. But this is still only a consistency result:

Theorem 3. If \clubsuit , there is a monotonically normal, locally compact, locally countable (hence first countable, and scattered) ω_1 -compact space of cardinality \aleph_1 which is not σ -countably compact.

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