1124-54-68 **Jerry E Vaughan*** (vaughanj@uncg.edu), University of North Carolina at Greensboro, Department of Mathematics and Statistics, 116 Petty Building, Greensboro, NC 27402-6170.
Companions of directed sets and the Ordering Lemma.

A well ordered set $(C, \preceq)$ is called a companion of a partially ordered set $(D, \leq)$ provided C is a cofinal subset of $(D, \leq)$ and for every $c_1, c_2 \in C$, if $c_1 \leq c_2$ then $c_1 \preceq c_2$. The ordering lemma is a version of the axiom of choice that says every directed set has a companion. If $f : D \to X$ is a net into a set X, the transfinite sequence $f \restriction C : (C, \preceq) \to X$ is called the companion (transfinte) sequence of the net f. We will give a proof of the Theorem: If $(D, \leq)$ does not have a well ordered cofinal subset (well ordered by the restriction of $\leq$) then there exists a topological space X and a net $f : D \to X$ such that the companion sequence $f \restriction C$ has a cluster point in X, but f does not have cluster point in X. This result points to a gap in the (now retracted) claimed proof by W. Sconyers and N. Howes that every normal linearly Lindleöf space is Lindelöf. (Received August 27, 2016)