Recently, the geometry of finite and infinite diamond graphs has been better understood. As is well known, the Lipschitz geometry of these graphs turned out to be fundamental regarding the dimension reduction problem in $\ell_1$. In this talk we will review what is known about the embeddability of the diamond graphs into classical, and more general, Banach spaces. In particular we will discuss the recent work of the speaker and his collaborators (R. Causey, S. Dilworth, D. Kutzarova, N. L. Randrianarivony, Th. Schlumprecht, S. Zhang) about the geometry of the countably branching diamond graphs, and the work of B. Randrianantoanina and M. I. Ostrovskii on the finitely branching diamond graphs. Some open problems will be mentioned. (Received March 12, 2017)