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Park Drive apt 17, Boston, MA 02215. *Sketching and Embedding are Equivalent for Norms.*

Imagine the following communication task. Alice and Bob each have a point from a metric space. They want to transmit a few bits and decide, whether their points are close to each other or are far apart. Of particular interest are sketching protocols: Alice and Bob both compute short summaries of their inputs and then a referee, given these summaries, makes the decision; sketches are very useful for various algorithms for massive datasets. Indyk (FOCS 2000) showed that for the  $\ell_p$  spaces with  $0 < p \leq 2$  the above problem allows a very efficient sketching protocol. Consequently, any metric that can be embedded into the  $\ell_p$  space with small distortion has a good protocol as well.

I will show that for normed spaces embedding into  $\ell_p$  is the only possible technique for solving the communication problem. Slightly more formally, we show that any normed space that admits a good communication (in particular, sketching) protocol for distinguishing close and far pairs of points embeds well into  $\ell_p$  with  $p$  being close to 1.

As a corollary, we will see communication lower bounds for the planar Earth Mover's Distance and for the trace norm by deriving them from the (known) non-embeddability theorems and (the contrapositive of) our result. (Received March 15, 2017)