1133-35-108 **Kazuo Yamazaki*** (kyamazak@ur.rochester.edu), Department of Mathematics, University of Rochester, Rochester, NY 14627. *On the Navier-Stokes equations in scaling-invariant spaces in any dimension.*

Whether the solution to the Navier-Stokes equations remains smooth for all time in a three-dimensional space remains a challenging open problem. In 1962 Serrin provided a certain space-time integrability condition for smoothness in a scaling-invariant norms for the weak solution to the Navier-Stokes equations, which is a three-dimensional velocity vector field. We discuss recent developments in the research direction in effort to improve such integrability conditions so that we only have to impose the condition on "only one of the three" velocity vector field components, instead of all of three. The proof crucially relies on a key identity which is a consequence of the divergence-free property, and techniques from anisotropic Littlewood-Paley theory that consists of anisotropic Bernstein's inequality, anisotropic Bony paraproducts and anisotropic Besov and Sobolev spaces. Moreover, except only a very few recent results by the speaker, all such results have been limited to the three-dimensional case; we will also discuss the progress toward extending to dimension such as four and beyond, such as the horizontal Biot-Savard law in higher dimensions. (Received July 16, 2017)