We investigate nonnegativity as an obstruction to various forms of structured completeness in $L^p$ spaces. For example, we prove that if each element of a system of functions in $L^p$ is pointwise nonnegative, then the system cannot be an unconditional basis or unconditional quasibasis (unconditional Schauder frame) for $L^p$. In particular, in $L^2$ this precludes the existence of nonnegative Riesz bases and frames. On the other hand, there exist pointwise nonnegative conditional quasibases in $L^p$, and there also exist pointwise nonnegative exact systems and Markushevich bases in $L^p$. (Received July 18, 2017)