The classical Paley-Wiener theorem says that an entire function of exponential type at most \( \pi \) whose restriction to the real axis is square-integrable is the Fourier transform of a function in \( L^2(-1/2, 1/2) \). We consider the following analogue: for a fixed singular measure \( \mu \) on \( [-1/2, 1/2] \), when is an entire function the Fourier transform of a function in \( L^2(\mu) \)? We give a complete characterization of such entire functions in terms of the Fourier transform of \( \mu \) as well as the Cauchy transform of \( \mu \). While the characterization is very different from the classical case—it does not involve any integrability conditions—we show that by reinterpreting the Paley-Wiener theorem, our characterization is in fact analogous. (Received July 11, 2017)