Identifying dense substructures is a frequent task in analyzing real-world graphs, with a rich history of results characterizing its computational complexity for various notions of substructure. For example, one can find the densest subgraph in polynomial time using flow-based methods, yet finding the densest clique or graph minor is NP-complete. We show that in some sense, finding dense substructures which are just slightly ‘less local’ than subgraphs seems to be intrinsically difficult.

Specifically, we consider $r$-shallow minors, which naturally intermediate between the local nature of subgraphs ($r = 0$) and the global notion of minors ($r = \infty$). Finding densest 0-shallow minors is in P, but Densest 1-Shallow Minor is NP-complete, so we focus on substructures that fall between 0- and 1-shallow. Specifically, we prove that Densest $r/2$-Shallow Topological Minor and Densest $r$-Subdivision are NP-complete already in sub-cubic apex-graphs for $r \geq 1$, and that neither problem can be solved in time $O(2^{o(n)})$ unless the Exponential Time Hypothesis (ETH) fails. Further, for Densest 1-Shallow Topological Minor, we show the problem is FPT for bounded treewidth, but no algorithm with running time $O(2^{o(tw(G)^2)}n)$ can exist unless the ETH fails. (Received July 28, 2017)