

1163-16-361

Aleks Kleyn* (aleks_kleyn@mailaps.org), aleks_kleyn@mailaps.org. *Calculus over quaternion algebra.*

The map $f : H \rightarrow H$ is called differentiable, if there exists differential form $\frac{df}{dx}$ such that

$$f(x+h) - f(x) = \frac{df}{dx} \circ h + o(h)$$

where

$$o : H \rightarrow H$$

is such continuous map that

$$\lim_{a \rightarrow 0} \frac{\|o(a)\|}{\|a\|} = 0$$

Linear map $\frac{df(x)}{dx}$ is called derivative of map f .

The differential form

$$\omega : H \rightarrow \mathcal{L}(D; H \rightarrow H)$$

is called integrable, if there exists a map

$$f : H \rightarrow H$$

such that

$$\frac{df(x)}{dx} = \omega(x)$$

Then we use notation

$$f(x) = \int \omega(x) \circ dx$$

and the map f is called indefinite integral of the differential form ω .

Let $U \subseteq A$ be open set. Let

$$\gamma : [a, b] \rightarrow U$$

be a path of class C^1 in U . We define the integral of the differential 1-form ω along the path γ by the equality

$$\int_{\gamma} \omega = \int_a^b dt \omega(\gamma(t)) \frac{d\gamma(t)}{dt}$$

Differential form

$$\omega : H \rightarrow \mathcal{L}(D; H \rightarrow H)$$

is integrable iff

$$d\omega(x) = 0$$

For any quaternions a, b , we define definite integral by the equality

$$\int_a^b \omega = \int_{\gamma} \omega$$

which does not depend on a path γ from a to b . (Received September 08, 2020)