Let $S$ be a smooth, complex surface and $F$ be a vector bundle. A result of Ellingsrud, Gottche, and Lehn says that any Chern number obtained using the tangent bundle of the Hilbert scheme of $n$ points on $S$ and the tautological sheaf associated to $F$ can be expressed as a polynomial (independent of $S$) in Chern numbers of $S$ and $F$. Even more structure is revealed when these numbers are assembled into certain generating series.

An example of such a number is the Euler characteristic of line bundles, which is of interest in strange duality. Strange duality relates the Euler characteristics of certain line bundles on moduli spaces. A strategy of Marian and Oprea relates these Euler characteristics to the cardinality of finite Quot schemes. In a recent paper, I have shown how viewing the expected length of the Quot scheme as a Chern number of a certain vector bundle suggests some surprising relationships between the generating series for these two sets of numbers. These conjectures have inspired some recent progress on computing top Segre classes of vector bundles on Hilbert schemes. (Received January 31, 2018)