Using the Kaczmarz algorithm, for any singular Borel probability measure \( \mu \) on the unit circle, we use a Parseval frame in \( L^2(\mu) \) “dextrodual” to the sequence \( \{e^{2\pi i nx}\}_{n=0}^{\infty} \) to construct positive matrices in \( H^2(\mathbb{D}) \) that have \( L^2(\mu) \) boundary function representations and are reproducing kernels with respect to those representations. For a given positive matrix \( K \) in \( H^2(\mathbb{D}) \) of a common form and a Borel measure \( \mu \) on the unit circle, we give a characterization of when \( K \) has \( L^2(\mu) \) boundary representations. This characterization is in terms of a matrix identity based on a new operator product called the Abel product. (Received February 01, 2018)