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Tsz Chiu Kwok* (tckwok0@gmail.com), **Lap Chi Lau**, **Yin Tat Lee** and **Akshay Ramachandran**. *The Paulsen Problem, Continuous Operator Scaling, and Smoothed Analysis*.

The Paulsen problem is a basic open problem in operator theory: Given vectors $u_1, \dots, u_n \in \mathbb{R}^d$ that are ϵ -nearly satisfying the Parseval's condition and the equal norm condition, is it close to a set of vectors $v_1, \dots, v_n \in \mathbb{R}^d$ that exactly satisfy the Parseval's condition and the equal norm condition? We consider the squared distance $\inf_v \sum_{i=1}^n \|u_i - v_i\|_2^2$ where the infimum is over the set of exact solutions. Previous results show that the squared distance of any ϵ -nearly solution is at most $O(\text{poly}(d, n, \epsilon))$ and there are ϵ -nearly solutions with squared distance at least $\Omega(d\epsilon)$. The fundamental open question is whether the squared distance can be independent of the number of vectors n .

We answer this question affirmatively by proving that the squared distance of any ϵ -nearly solution is $O(d^{13/2}\epsilon)$. Our approach is based on a continuous version of the operator scaling algorithm. We first define a dynamical system based on operator scaling to give a looser bound, and then we show that the dynamical system will converge faster by slightly perturbing the input vectors. (Received February 05, 2018)