The Paulsen problem is a basic open problem in operator theory: Given vectors $u_1, \ldots, u_n \in \mathbb{R}^d$ that are $\epsilon$-nearly satisfying the Parseval’s condition and the equal norm condition, is it close to a set of vectors $v_1, \ldots, v_n \in \mathbb{R}^d$ that exactly satisfy the Parseval’s condition and the equal norm condition? We consider the squared distance $\inf_v \sum_{i=1}^{n} \|u_i - v_i\|_2^2$ where the infimum is over the set of exact solutions. Previous results show that the squared distance of any $\epsilon$-nearly solution is at most $O(poly(d, n, \epsilon))$ and there are $\epsilon$-nearly solutions with squared distance at least $\Omega(d\epsilon)$. The fundamental open question is whether the squared distance can be independent of the number of vectors $n$.

We answer this question affirmatively by proving that the squared distance of any $\epsilon$-nearly solution is $O(d^{13/2}\epsilon)$. Our approach is based on a continuous version of the operator scaling algorithm. We first define a dynamical system based on operator scaling to give a looser bound, and then we show that the dynamical system will converge faster by slightly perturbing the input vectors. (Received February 05, 2018)