Given two Riemannian manifolds with positive Ricci curvature, is it possible to find a metric on the connected sum that also has positive Ricci curvature? The general approach to questions of this type, following the Gromov-Lawson construction, is to find a local deformation of the metric so that the resulting metric is standard near a point and still has the desired curvature conditions. The Gromov-Lawson construction applied directly can be seen to leave positive Ricci curvature, and the source of this failure is that the choice of “standard” necessarily forces negative curvature.

In this talk, we will revisit a paper of Perelman, in which he constructs metrics with positive Ricci curvature on the connected sum of arbitrarily many complex projective planes. I will explain my recent work that generalizes this construction to metrics with positive Ricci curvature on arbitrary connected sums of complex, quaternionic, and octonionic projective spaces in every dimension. I will also explain Perelman’s candidate for what “standard metric” we ought to use when considering a Gromov-Lawson approach to this problem. He showed that a local deformation exists in very special circumstance, to which I will add some further examples. (Received January 29, 2018)