The Kreck-Stolz $s$ invariant is used to distinguish connected components of the moduli space of positive scalar curvature metrics. We use a formula of Kreck and Stolz to calculate the $s$ invariant for metrics on $S^n$ bundles with nonnegative sectional curvature. We then apply it to show that the moduli spaces of metrics with nonnegative sectional curvature on certain 7-manifolds have infinitely many path components. These include the first non-homogeneous examples of this type and certain positively curved Eschenburg and Aloff-Wallach spaces. (Received February 03, 2018)