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**Pavel Galashin, Sam Hopkins\*** (shopkins@mit.edu), **Thomas McConville** and **Alex Postnikov**. *Vector-firing for root systems*.

Hopkins, McConville and Propp recently introduced a variant of chip-firing on the infinite path where the chips are given distinct integer labels and showed this process sorts certain (but not all) initial configurations of chips. We recast this result in terms of root systems: the labeled chip-firing game can be seen as a “vector-firing” process which allows the moves  $\lambda \rightarrow \lambda + \alpha$  for  $\alpha \in \Phi^+$  whenever  $\langle \lambda, \alpha^\vee \rangle = 0$ , where  $\Phi^+$  is the set of positive roots of a root system of type  $A_{2n-1}$ . We give conjectures about confluence for this process in the general setting of an arbitrary root system. We show that the process is always confluent from any initial point after modding out by the action of the Weyl group (an analog of unlabeled chip-firing in arbitrary type). We also show that if we instead allow firing when  $\langle \lambda, \alpha^\vee \rangle \in [-k-1, k-1]$  or  $[-k, k-1]$ , we always get confluence from any initial point. Moreover, in these two settings, the set of weights with given stabilization has a remarkable geometric structure related to permutohedron. This geometric structure leads us to define certain “Ehrhart-like” polynomials that conjecturally have nonnegative integer coefficients. (Received July 16, 2017)