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Manuel Cortés-Izurdiaga* (mizurdia@ual.es). *Maximal ideals in module categories.*

An ideal \mathcal{I} in a preadditive category \mathbf{C} is an additive subfunctor of the Hom bifunctor. As in the case of rings, one can consider maximal and minimal ideals in the category \mathbf{C} . We are interested in maximal ideals in the category $\text{Mod-}R$ of modules over a ring R .

While minimal ideals in $\text{Mod-}R$ are well understood, as a consequence of a result, proved by A. Facchini, which establishes a one-to-one correspondence between minimal ideals in $\text{Mod-}R$ and simple modules, there is no such description of maximal ideals. The main goal of the talk is to prove that actually there do not exist maximal ideals in $\text{Mod-}R$ (in fact, there do not exist in Grothendieck categories).

The main idea to prove this result is to relate the order inclusion between ideals in \mathbf{C} with a new preorder, \preceq , between objects in the category, in such a way that the existence of maximal ideals implies the existence of maximal objects with respect to \preceq . Now, noting that big direct sums of copies of a module M are greater than M with respect to \preceq , we conclude that there do not exist maximal ideals in the category of modules over a ring.

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