In [S. Fujita, G. MacGillivray, T. Sakuma: Safe set problem on graphs. Discrete Applied Math. 215: 106-111 (2016)], the authors defined a safe set in a graph $G = (V(G), E(G))$ as a set $S$ of vertices of $G$ with the property that $|V(C)| \geq |V(D)|$ for every component $C$ of the subgraph $G[S]$ of $G$ induced by $S$ and every component $D$ of the subgraph $G - S$ of $G$ induced by $V(G) \setminus S$ such that some vertex in $C$ is adjacent to some vertex in $D$. For convenience, we call two disjoint subgraphs $C$ and $D$ of $G$ adjacent if some vertex in $C$ is adjacent to some vertex in $D$.

We can naturally extend this notion to the “weighted version”. For a graph $G$ and a weight function $w : V(G) \to \mathbb{Z}_{\geq 0}$, we consider the vertex weighted graph $(G, w)$. For a set $U$ of vertices of $G$, let $w(U) = \sum_{u \in U} w(u)$. A set $S$ of vertices of $G$ is a weighted safe set in $G$ if $w(C) \geq w(D)$ for every component $C$ of $G[S]$ and every component $D$ of $G - S$ such that $D$ is adjacent to $C$. For a given $(G, w)$, what is the smallest cardinality of a weighted safe set?

In this talk, I would like to give a short survey on the weighted safe set problems in vertex-weighted graphs. (Received January 14, 2019)