In this talk, we consider the \((n + 1)\)-variable combinatorial rank generating function \(R_n(x_1, x_2, \ldots, x_n; q)\) for \(n\)-marked Durfee symbols. These are \(n + 1\) dimensional multisums for \(n > 1\), and specialize to the ordinary two-variable partition rank generating function when \(n = 1\). The mock modular properties of \(R_n\) when viewed as a function of \(\tau \in \mathbb{H}\), with \(q = e^{2\pi i \tau}\), for various \(n\) and fixed parameters \(x_1, x_2, \ldots, x_n\), have been previously studied. Namely, Bringmann and Ono considered when \(n = 1\) and \(x_1\) a root of unity; Bringmann considered when \(n = 2\) and \(x_1 = x_2 = 1\); Bringmann, Garvan, and Mahlburg considered when \(n \geq 2\) and \(x_1 = x_2 = \cdots = x_n = 1\); and Folsom and Kimport considered when \(n \geq 2\) and the \(x_j\) are suitable roots of unity \((1 \leq j \leq n)\).

The quantum modular properties of \(R_1\) follow from existing results. In our work, we prove for any \(n \geq 2\) that the combinatorial generating function \(R_n\) is a quantum modular form when viewed as a function of \(x \in \mathbb{Q}\), where \(q = e^{2\pi i x}\), and the \(x_j\) are roots of unity.

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