Necklace polynomials arise naturally in combinatorics, number theory, dynamics, and representation theory. It was recently observed that necklace polynomials vanish at many roots of unity. I will demonstrate this phenomenon through examples and discuss how it leads to surprising connections with geometry, arithmetic, and group theory. (Received January 24, 2019)